Department of Mechanical, Materials & Manufacturing Engineering

DYNAMICS (VIBRATION)

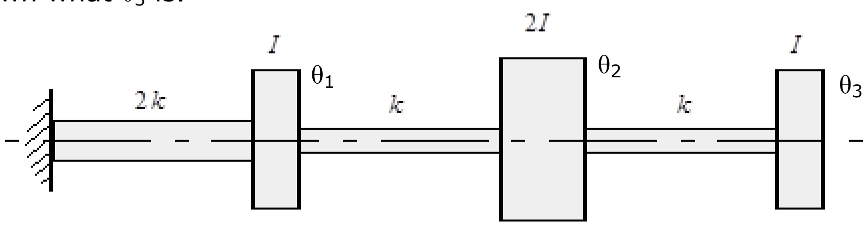
SHEET 6 : APPROXIMATE METHODS

Take E = 207 GPa, ρ = 7800 kg/m^2

1. Use Rayleigh's Method to estimate the lowest natural frequency of the torsional system shown below.

   Guess (A): Assume all deflections are in phase. E.g. any set of values for which $\theta_1 < \theta_2 < \theta_3$

   Guess (B): The "static deflection shape" can be found by assuming that a static torque is applied to each disc that is proportional to its moment of inertia. E.g. $\theta_1 : \theta_2 : \theta_3 = 2 : 5 : ??$; where you should be able to figure out on your own what θ_3 is.



   Answer: The value depends on the assumed mode shape, but will be greater than the exact value of $0.445\sqrt{k / I}$

2. A shaft with universal joints at each end has a length of 6 m, a second moment of area of 0.00025 m^4 and a mass/unit length of 75 kg/m. It carries three discs, which can be regarded as point masses of 100, 150, and 200 kg located 1.2, 3 and 4.8 m from the left-hand end. Estimate the lowest critical speed by the following methods.

   (i) Rayleigh's method, assuming an appropriate trigonometrical form for the mode shape and neglecting the mass of the shaft,

   (ii) as for (*iii*), but including the mass of the shaft.

   Answer: Values for (*i*) and (*ii*) depend on the mode shape but are likely to be within 5% of (*i*) *1960 rev/min* and *(ii) 1456 rev/min*.

3. (a) Using the concepts of dynamically equivalent systems, estimate values of m^+ and k^+ for the approximate model in Figure Q.3(b) based on an equivalence with the motion of the upper mass in Figure Q.3(a). Use the values of m^+ and k^+ to estimate the lowest natural frequency of the system.

(b) Use your approximate model to estimate the steady-state response of the upper mass to a sinusoidal force of frequency ω and amplitude P applied to it.

(c) Use your answer to (b) together with your assumed mode shape to estimate the response of the *lower* mass.

Figure Q.3 (a) (b)

Answers to Q.3(a) depend on the mode shape assumed. The exact mode shape gives $m^+ = 3.164\ m$, $k^+ = 1.682\ k$ and $\omega_n = 0.7291\sqrt{k/m}$ (which is also the exact lowest natural frequency). A good estimate of the mode shape should get you to within a few percent of these figures.